Model Answer (Electromagnetic Field Theory)
B. Tech. Fifth Semester (ECE), Session, 2013-14 (ODD Semester)

AS-4149

B.Tech. V semester, Exam- 2013, ECE

Electromagnetic Field Theory

(ECE 3102)

Section-'A'

Ans.1.

(i) $\sin\theta$, 0

(ii) The Biot-Savart law states that at any point 'P' the magnitude of magnetic field intensity produced by the differential element is proportional to the product of current, the magnitude of differential length and the sine of angle between the filament and the line connecting differential length to point P. The magnitude of magnetic field intensity is inversely proportional to the square of distance from the differential element to the point P.

$$dH = \frac{I\,dl \sin\theta}{4\pi R^2}\hat{a}_R$$

(iii) $D_{N_1} - D_{N_2} = \rho_s$, $H_{t_1} - H_{t_2} = K$

(iv) $\gamma = \sqrt{j\omega\mu\sigma}$, $\eta = \sqrt{j\omega\mu/\sigma}$

(v) 0, 1

(vi) $$\nabla A = \frac{\partial A}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial A}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial A}{\partial \phi}\hat{a}_\phi$$

$$\nabla \times V = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(H_\phi \sin\theta) - \frac{\partial}{\partial\phi}H_\theta\right]\hat{a}_r +$$

$$\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}H_r - \frac{\partial}{\partial r}(rH_\phi)\right]\hat{a}_\theta + \frac{1}{r}\left[\frac{\partial}{\partial r}(rH_\theta) - \frac{\partial}{\partial\theta}H_r\right]\hat{a}_\phi$$

(vii) $$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial\phi^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial V}{\partial\theta}\right] + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2} = 0$$

(viii) $\nabla \times \vec{E} = 0$

$\nabla \times \vec{H} = 0$

$\nabla \cdot \vec{D} = 0$

$\nabla \cdot \vec{B} = 0$

(ix) $\nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} + \mu\sigma \frac{\partial \vec{E}}{\partial t}$

$\nabla^2 \vec{B} = \mu\epsilon \frac{\partial^2 \vec{B}}{\partial t^2} + \mu\sigma \frac{\partial \vec{B}}{\partial t}$

(x) $Z_0 = \sqrt{\frac{R}{j\omega C}}$, $\gamma = \sqrt{j\omega RC}$

Section - 'B'

Ans. 2 ⇒ Vector transformation between Cartesian and cylindrical co-ordinates -

let

$$\vec{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$$

Here $\hat{a}_x$, $\hat{a}_y$ and $\hat{a}_z$ are unit vectors in X, Y and Z-directions respectively in cartesian co-ordinate system. The same vector $\vec{A}$ can be expressed in cylindrical coordinate system as-

$$A = A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \hat{a}_z$$

Here $\hat{a}_\rho$, $\hat{a}_\phi$ and $\hat{a}_z$ are unit vectors in cylindrical coordinate system.

so

$$A_\rho = \vec{A} \cdot \hat{a}_\rho$$

$$A_\rho = (A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z) \cdot \hat{a}_\rho$$

$$A_\rho = A_x(\hat{a}_x \cdot \hat{a}_\rho) + A_y(\hat{a}_y \cdot \hat{a}_\rho) + A_z(\hat{a}_z \cdot \hat{a}_\rho)$$

$$= A_x \cos\phi + A_y \sin\phi + 0$$

$$\Rightarrow A_\rho = A_x \cos\phi + A_y \sin\phi$$

Simillarly $A_\phi = -A_x \sin\phi + A_y \cos\phi$

$A_z = A_z$

In the same method -

$$A_x = (A)\cdot\hat{a}_x$$

$$\Rightarrow A_x = (A_\rho\hat{a}_\rho + A_\phi\hat{a}_\phi + A_z\hat{a}_z)\cdot\hat{a}_x$$

$$\Rightarrow A_x = A_\rho(\hat{a}_\rho\cdot\hat{a}_x) + A_\phi(\hat{a}_\phi\cdot\hat{a}_x) + A_z(\hat{a}_z\cdot\hat{a}_x)$$

$$\Rightarrow A_x = A_\rho\cos\phi - A_\phi\sin\phi + A_z\cdot 0$$

$$\Rightarrow A_x = A_\rho\cos\phi - A_\phi\sin\phi$$

Simillarly $A_y = A_\rho\sin\phi + A_\phi\cos\phi$

$$A_z = A_z.$$

Ans. 3 $\Rightarrow$ (i) $U = x^2y + xyz$

As gradient of any scalar field in cartesian co-ordinate system- is given as -

$$\nabla U = \frac{\partial U}{\partial x}\hat{a}_x + \frac{\partial U}{\partial y}\hat{a}_y + \frac{\partial U}{\partial z}\hat{a}_z$$

so

$$\frac{\partial U}{\partial x} = \frac{\partial}{\partial x}(x^2y + xyz)$$

$$= 2xy + yz$$

$$\frac{\partial U}{\partial y} = \frac{\partial}{\partial y}(x^2y + xyz)$$

$$= x^2 + xz$$

$$\frac{\partial U}{\partial z} = \frac{\partial}{\partial z}(x^2y + xyz)$$

$$= xy$$

so $\nabla U = y(2x+z)\hat{a}_x + x(x+z)\hat{a}_y + xy\hat{a}_z$

(ii) $V = \rho z\sin\phi + z^2\cos^2\phi + \rho^2$

As gradient of any scalar field in cylindri

-cal coordinate system is given as -

$$\nabla V = \frac{\partial V}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a}_\phi + \frac{\partial V}{\partial z}\hat{a}_z$$

$$\frac{\partial V}{\partial \rho} = \frac{\partial}{\partial \rho}(\rho z \sin\phi + z^2\cos^2\phi + \rho^2)$$

$$= (z\sin\phi + 2\rho)$$

$$\frac{1}{\rho}\frac{\partial V}{\partial \phi} = \frac{1}{\rho}\frac{\partial}{\partial \phi}(\rho z\sin\phi + z^2\cos^2\phi + \rho^2)$$

$$= \left(z\cos\phi - \frac{z^2}{\rho}\sin 2\phi\right)$$

$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}(\rho z\sin\phi + z^2\cos^2\phi + \rho^2)$$

$$= (\rho\sin\phi + 2z\cos^2\phi)$$

So

$$\nabla V = (z\sin\phi + 2\rho)\hat{a}_\rho + \left(z\cos\phi - \frac{z^2}{\rho}\sin 2\phi\right)\hat{a}_\phi + (\rho\sin\phi + 2z\cos^2\phi)\hat{a}_z.$$

Ans. 4 ⇒ Magnetic Dipole ⇒ The magnetic field produced by a small current loop is simillar to the electric field from a small electric dipole. For this reason a small current loop is called magnetic dipole. Its dipole moment P_m is defined as equal to the product of the area of the plane loop and the magnitude of circulating current. The vector direction of the moment is perpendicular to the plane of the loop and is along the direction of a right hand screw, moved in the direction of current in the loop.

a) Energy stored in Electrostatic field:

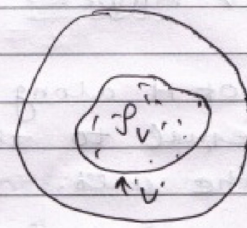
Let a volume 'V' containing a charge of volume charge density ρ_v is enclosed within a large sphere of radius R, ρ_v vanishes outside 'V'.

so as -

$$W_e = \frac{1}{2}\int_v \rho_v \cdot V \, dv$$

$$\Rightarrow W_e = \frac{1}{2}\int (\nabla \cdot D) \cdot V \, dv \qquad \because \nabla \cdot D = \rho_v$$

$$\because \nabla \cdot (V \cdot D) = V(\nabla \cdot D) + D \cdot (\nabla V)$$



So.

$$W_e = \frac{1}{2}\int_v (\nabla \cdot VD) dv - \frac{1}{2}\int_v (D \cdot \nabla V) \, dv$$

because $\frac{1}{2}\int_v (\nabla \cdot VD) \, dv = \frac{1}{2}\oint_s \cancel{\nabla D} (VD) \cdot ds$

$\because$ (Gauss divergence)

For a very large sphere, the volume charge looks like a point charge. At very large R, $D \propto \frac{1}{R^2}$, $V \propto \frac{1}{R}$ so $VD \propto \frac{1}{R^3}$ and $S \propto R^2$, so

$$\frac{1}{2}\oint_{R\to\infty} (VD) \cdot ds = 0$$

so $W_e = -\frac{1}{2}\int_v (D \cdot \nabla V) \, dv$

$$\Rightarrow W_e = \frac{1}{2}\int_v (D \cdot E) \, dv \qquad \because (E = -\nabla V)$$

$$\boxed{W_e = \frac{1}{2}\int_v \epsilon_0 E^2 \, dv}$$

By anology with electric field, the expression for total energy stored in a steady magnetic field in which B is linearly related to H is -

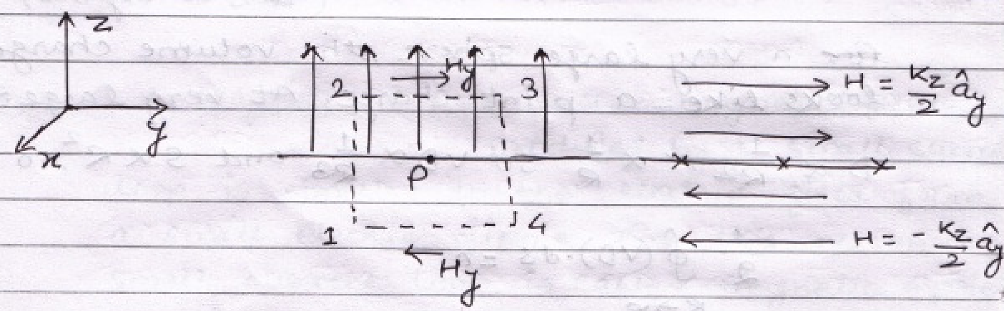
$$W_m = \frac{1}{2}\int_V B.H\, dv$$

$$W_m = \frac{1}{2}\int_V \mu H^2 dV = \frac{1}{2}\int_V \frac{B^2}{\mu}\, dv$$

Ans. 5 ⇒ Ampere's Circuital law ⇒ It states that The line integral of H along any closed path is exactly equal to the direct current enclosed by the path.

$$\oint H \cdot dl = I_{encl}$$

A plane sheet carrying current of density K_z in z-direction is shown in fig.



The field can be thought of considering sheet as current elements. Field shall not vary with respect to z, $H_z = 0$. It will not vary with respect to x, due to symmetry. It can vary with respect to y axis only. consider a path 1-2-3-4-1 with centre P, each side as '2a'. No field will exist in 1-2 or 3-4 direction. The total line integral of H =

$$\oint H\cdot dl = \int_1^2 H\cdot dl + \int_2^3 H\cdot dl + \int_3^4 H\cdot dl + \int_4^1 H\cdot dl$$

$$= I_{enclo.}$$

$$\oint H\cdot dl = 0 + H_y(2a) + 0 + H_y(2a) = K_z\cdot 2a$$

$$4aH_y = K_z\cdot 2a$$

$$\Rightarrow H_y = \frac{K_z}{2}\hat{a}_y \quad \text{(above the sheet)}$$

So $\boxed{H = \frac{K}{2}\hat{a}_N}$

where $\hat{a}_N$ is normal to surface. The field above and below the plane shall be $\frac{K_z}{2}\hat{a}_y$ and $-\frac{K}{2}\hat{a}_y$ respectively.

Ans. 6 ⇒ The given value of $\vec{E}$ is

$$\tilde{E}(z,t) = 50\cos(\omega t - \beta z)\,\hat{a}_x$$

we have the standard equation of E is-

$$\vec{E} = E_{ox}\cos(\omega t - \beta z)\,\hat{a}_x$$

$$E_{ox} = 50$$

Here E is in X-direction. So $\vec{H}$ will be in Y-direction. $\vec{E}$ and $\vec{H}$ are related as-

$$\vec{E} = \eta\vec{H}$$

$$\vec{H} = \frac{\vec{E}}{\eta}, \quad \eta = 120\pi \text{ (for freespace)}$$

$$\vec{H} = \frac{50}{120\pi}\cos(\omega t - \beta z)\,\hat{a}_y$$

$$\vec{H} = 0.13\cos(\omega t - \beta z)\,\hat{a}_y$$

The poynting vector is given by

$$P = \vec{E} \times \vec{H}.$$

$$\vec{P} = [50 \cos(\omega t - \beta z)\,\hat{a}_x] \times [0.13 \cos(\omega t - \beta z)\,\hat{a}_y]$$

$$\vec{P} = 6.5 \cos^2(\omega t - \beta z)\,\hat{a}_z$$

Eqn' indicates that the power flow is in the z-direction. Now the net power flow in the z-direction is -

$$P_z = \frac{E_{ox}^2}{2\eta} \times S$$

$$= \frac{50^2}{2 \times 120\pi} \times \pi \times (2.5)^2$$

$$P_z = 65.1 \text{ watts.}$$

Ans. 7 ⇒ The continuity equation of current is based on the principle of conservation of charge. It states that the electric charges may not be created or destroyed.

Consider an arbitrary volume 'V' bounded by surface 'S'. A net charge 'Q' exists within this region. Then if the current flows out of this region, the charge in the volume decreases. This rate of decrease is equal to the outgoing rate of current. Similarly if the current flows into this region, the charge in the volume must increase.

By considering this principle of conservation of charge, there is two forms of this continuity equation

i) Integral form

ii) Point form.

Integral form ⇒ consider any arbitrary volume 'V' bounded by the surface 'S'. Now the outward current from this closed surface is obtained by taking the closed surface integration of current density ($\vec{J}$)

$$\therefore \oint_S \vec{J} \cdot d\vec{s} = I$$

Mathematically, the current coming out from the volume V is represented as a rate of decrease of current in that volume.

Thus $I = \frac{-dQ_i}{dt}$

So $$\boxed{I = \oint_S J \cdot ds = \frac{-dQ_i}{dt}}$$

Point form ⇒ Using divergence theorem for the closed surface integration

$$\oint_S \bar{J} \cdot d\bar{s} = \int_V (\nabla \cdot J)\, dv$$

So
$$\int_V (\nabla \cdot J)\, dv = \frac{-dQ_i}{dt}$$

Because
$$Q_i = \int_V \rho_v \cdot dv$$
ρ_v = volume charge density..

$$\int_V (\nabla \cdot J)\, dv = -\frac{d}{dt} \int_V \rho_v \cdot dv$$

Here the surface area is constant so -

$$\boxed{\nabla \cdot J = \frac{-\partial \rho_v}{\partial t}}$$

Ans. 8 ⇒ Consider that a plane wave is propagating in the positive z-direction. The electric field intensity is in x-direction & magnetic field intensity is in Y-direction. We can write the solution of this wave equation in terms of $\bar{E}$ as-

$$\bar{E} = E_0 e^{-\gamma z} \hat{a}_x \quad \text{—— (1)}$$

from maxwell's equation-

$$\frac{\partial E_y}{\partial z} = j\omega\mu H_x \quad \text{—— (2)}$$

and $$\frac{\partial E_x}{\partial z} = -j\omega\mu H_y \quad \text{—— (3)}$$

and from eqn (1).

$$\frac{\partial E}{\partial z} = -\gamma E_0 e^{-\gamma z}$$

$$\frac{\partial E_x}{\partial z} = -\gamma E_x$$

So $-\gamma E_x = -j\omega\mu H_y$

$$\Rightarrow \frac{E_x}{H_y} = \frac{j\omega\mu}{\gamma} = \sqrt{\frac{j\omega\mu}{(\sigma + j\omega\epsilon)}} = \eta$$

$\underline{\underline{\alpha}}$ ⇒ $\because \gamma^2 = j\omega\mu\sigma + j^2\omega^2\mu\epsilon$

and $\gamma^2 = (\alpha + j\beta)^2$

$$= \alpha^2 - \beta^2 + 2j\alpha\beta$$

So $\alpha^2 - \beta^2 = -\omega^2\mu\epsilon$ ←— (4)

& $2\alpha\beta = \omega\mu\sigma$ —— (5)

By using eqn. (4) & (5)

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}}-1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}}+1\right]}$$

and
$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}}+1\right]}}$$

Ans. 9 ⇒

1) Skin depth : copper is a good conductor. so skin depth is given by.

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} \quad \text{and } \omega = 2\pi f$$

$$\delta = \sqrt{\frac{2}{2\pi\times 100\times 10^6\times 4\pi\times 10^{-7}\times 58\times 10^6}}$$

$$\therefore \boxed{\delta = 6.608\ \mu m}$$

ii) Intrinsic impedence ⇒

$$\eta = \sqrt{\frac{\omega\mu}{2\sigma}}\,(1+j)$$

$$\Rightarrow \eta = \sqrt{\frac{2\pi f\mu}{2\sigma}}\,(1.414\angle 45^\circ)$$

$$\Rightarrow \eta = (2.608\times 10^{-3}\angle 0^\circ)(1.414\angle 45^\circ)$$

$$\Rightarrow \eta = 3.689\times 10^{-3}\angle 45^\circ$$

$$\boxed{\eta = 2.608\times 10^{-3} + j\,2.608\times 10^{-3}\ \Omega}$$

Ans. 10 $\Rightarrow$ $\because \gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$

$$\gamma = \sqrt[4]{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} \angle \frac{1}{2}\left[\tan^{-1}\frac{\omega L}{R} + \tan^{-1}\frac{\omega C}{G}\right]$$

But $|\gamma| = \sqrt[4]{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}$

and $\gamma = \alpha + j\beta$

$$|\gamma| = \sqrt{\alpha^2+\beta^2}$$

So

$$\sqrt{\alpha^2+\beta^2} = \sqrt[4]{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}$$

$$\alpha^2+\beta^2 = \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} \quad \text{—— (1)}$$

and because

$$\alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$(\alpha+j\beta)^2 = (R+j\omega L)(G+j\omega C)$$

$$\alpha^2 + 2j\alpha\beta - \beta^2 = RG + j\omega RC + j\omega LG - \omega^2 LC$$

so $\alpha^2 - \beta^2 = RG - \omega^2 LC$ —— (2)

and $2j\alpha\beta = j\omega(RC + LG)$ —— (3)

from eqn (1) & (2)

$$\alpha = \sqrt{\frac{1}{2}\left[(RG - \omega^2LC) + \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right]}$$

& $$\beta = \sqrt{\frac{1}{2}\left[\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} - (RG - \omega^2LC)\right]}$$

Distortion less $\Rightarrow$ $LG = RC$

for UG Cable. $\alpha = \beta = \sqrt{\frac{\omega RC}{2}}$

Ans. 11 ⇒

Because -

$$Z_0 = \sqrt{\frac{Z}{Y}} \quad \& \ \gamma = \sqrt{ZY}$$

$$Z_0 \cdot \gamma = Z$$

$$\Rightarrow Z = R + j\omega L = 2040(0.054\angle 87.9^\circ)$$

$$= 110.16\angle 87.9^\circ$$

$$R + j\omega L = 4.03 + j110.086$$

So $R = 4.03\ \Omega/km.$

$j\omega L = j\,111.086$

$L = 0.0219\ H/km.$

Similarly

$$\frac{\gamma}{Z_0} = Y$$

$$\Rightarrow G + j\omega C = \frac{0.054\angle 87.9^\circ}{2040}$$

$$G + j\omega C = 2.647\times10^{-5}\angle 87.9'$$

$$G + j\omega C = (9.7\times10^{-7}) + j\,2.645\times10^{-5}$$

$$\Rightarrow G = 9.7\times10^{-7}\ mho/km.$$

$$\omega C = 2.645\times10^{-5}$$

$$C = 5.262\times10^{-9}\ F/km.$$

So
$$\boxed{\begin{aligned} R &= 4.03\ \Omega/km \\ L &= 0.0219\ H/km \\ G &= 9.7\times10^{-7}\ mho/km \\ C &= 5.262\ nF/km \end{aligned}}$$